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Abstract: This paper proposes developing a H_{∞} dynamic output-feedback decentralized control design method for nonlinear interconnected systems subject to time-varying parameters and external disturbances. The designed controller is formulated as an optimization problem subject to linear matrix inequalities (LMIs) for the concurrent computation of the decentralized observation and control gains, and for the external disturbance mitigation by means of a H_{∞} performance criterion minimization. The propounded optimization problem, designed in LMI conditions, is expeditiously resolved by a one-step procedure to override the conservatism generated by using many step-based procedures often used in the analysis and synthesis of interconnected systems. The effectiveness of the developed control scheme is demonstrated through simulation results of multimachine power systems.

Keywords: Decentralized observation and control, H_{∞} criterion, interconnected systems, LMI optimization, multimachine power systems.

1. INTRODUCTION

In the last few years, a significant progress has been realized in divers control domains of interconnected processes. This progress is motivated by emerging applications of novel actuation devices for cooperating robotic systems [1], aerospace processes [2], inverted pendulums [3] and power systems [4]. Therefore, designing a centralized controller for interconnected processes may not be rigorous because of high dimensions, time multiscale and geographic distribution of subsystems. Furthermore, the interconnections between subsystems are always non-linear with parameter variations and affected by exogenous disturbances. These constraints motivate the design of decentralized control schemes with no communication transfer among individual controllers leading to practicable technically control methodologies [5].

In the state-feedback control design, all the state variables are always difficult to reach, not wholly available or costly to measure with accurate sensors. It is then appropriate to synthesize a state observer able to estimate the unavailable states that can be used in other applications such as the fault detection and isolation. Recently, many works are dealt with this problem. In [6], for instance, it is investigated the problem of fault detection filtering for nonlinear switched stochastic systems in the T-S fuzzy framework. While, the work in [7] is devoted to tackle the problem of Hankel-norm output-feedback controller design for a class of T-S fuzzy stochastic systems. Thus,

the fuzzy-basis-dependant output-feedback controller design method is analyzed with the aid of fuzzy Lyapunov function technique. On the other hand, the authors in [8] addressed the stability analysis and the robust stabilization of nonlinear interconnected systems through the use of H_{∞} criterion. Thus, the designed method is characterized in the LMI framework. Nevertheless, my paper deals with the problem of robustness design of a H_{∞} decentralized output-feedback model reference tracking controller for a class of large-scale nonlinear systems.

As a matter of fact, the developed control approach in this work is propounded to guarantee the asymptotic stability in the Lyapunov framework and the model reference tracking control, to reconstruct the unavailable state variables, and to ensure the robust performances by mitigating the exogenous disturbances applied to the overall interconnected system. The novelty of this article is specifically the LMI formulation that can be applied for the decentralized robust control design of nonlinear uncertain disturbed interconnected systems. Thereby, both the Lyapunov quadratic stability and the H_{∞} performances of the studied system are proposed as an innovative optimization problem subject to LMI constraints, which can be solved by a one-step LMI optimization design to compute concurrently the control and observation gain matrices. However, several optimization problems, dedicated to controlling interconnected systems, are formulated as bilinear matrix inequalities (BMIs) and solved by several step-based procedures that can calculate separately the

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control and the observation gains leading to conservative and suboptimal solutions [9]. Besides, many optimization problems are resolved by using algebraic Riccati equations, which may be difficult to resolve and can not readily cover diverse types of further constraints [10].

The reminder of this work is presented in the following order. The system description and the problem under investigation are stated in Section 2. In Section 3, the H_{∞} decentralized observation and control design is proposed as an optimization problem subject to LMI constraints, while Section 4 highlights the simulation results of the developed control approach applied to a power system with three interconnected generators. At last, some conclusions end this work.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A large-scale interconnected system composed of *N* subsystems with uncertain parameters and external disturbances can be described by

$$\begin{cases} \dot{x}_{i}(t) = (A_{i} + \Delta A_{i})x_{i}(t) + (B_{i} + \Delta B_{i})u_{i}(t) \\ + \xi_{i}(t, x(t)) + \theta_{i}(t) \\ y_{i}(t) = C_{i}x_{i}(t), \quad i = 1, \dots, N, \end{cases}$$
(1)

where $A_i \in \mathbb{R}^{n_i \times n_i}$ is the state matrix, $B_i \in \mathbb{R}^{n_i \times m_i}$ is the control matrix and $C_i \in \mathbb{R}^{p_i \times n_i}$ represents the output matrix of each subsystem; $\Delta A_i \in \mathbb{R}^{n_i \times m_i}$ and $\Delta B_i \in \mathbb{R}^{n_i \times m_i}$ are time-varying matrices representing norm-bounded parametric uncertainties as follows:

$$[\Delta A_i \ \Delta B_i] = D_i F_i(t) [E_{1i} \ E_{2i}], \ F_i^T(t) F_i(t) \le I$$
(2)

with D_i , E_{1i} and E_{2i} known constant matrices of suitable dimensions. $F_i(t)$ reflects an unknown function assumed to have Lebesgue measurable elements and I represents the identity matrix of a suitable size.

 $x_i(t) \in \mathbb{R}^{n_i}, u_i(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^{p_i}$ designate the state vector, the control vector and the output vector of the *i*th subsystem, respectively; $\theta_i(t)$ is the external disturbance vector and $\xi_i(t, x(t))$ denotes the nonlinear interconnection function vector, which is supposed to be uncertain and meet the following inequality:

$$\|\xi_i(t, x(t))\| \le \alpha_i \|x(t)\| \tag{3}$$

with $\alpha_i > 0$ (i = 1, ..., N) known interconnection real constant bounds and $\|.\|$ the Euclidean norm.

The state equation of the global system is expressed by

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ + \xi(t, x(t)) + \theta(t), \\ y(t) = Cx(t), \end{cases}$$
(4)

with

$$\begin{aligned} x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_N(t) \end{bmatrix}, \\ y(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix}, \quad \xi(t, x(t)) = \begin{bmatrix} \xi_1(t, x(t)) \\ \xi_2(t, x(t)) \\ \vdots \\ \xi_N(t, x(t)) \end{bmatrix}, \\ \theta(t) &= \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \vdots \\ \theta_N(t) \end{bmatrix}, \quad A = \operatorname{diag}\{A_i\}, \quad B = \operatorname{diag}\{B_i\}, \end{aligned}$$

and $C = \text{diag}\{C_i\}$.

The uncertainty matrices ΔA and ΔB are defined in the following form:

$$[\Delta A \quad \Delta B] = DF(t)[E_1 \quad E_2], \quad F^T(t)F(t) \le I \qquad (5)$$

with $D = \text{diag}\{D_i\}$, $F(t) = \text{diag}\{F_i(t)\}$, $E_1 = \text{diag}\{E_{1i}\}$ and $E_2 = \text{diag}\{E_{2i}\}$ for i = 1, ..., N.

The state observer of each subsystem, using only local information of the subsystem inputs and outputs, is expressed by

$$\begin{cases} \dot{\hat{x}}_{i}(t) = A_{i}\hat{x}_{i}(t) + B_{i}u_{i}(t) + L_{i}[y_{i}(t) - \hat{y}_{i}(t)]\\ \hat{y}_{i}(t) = C_{i}\hat{x}_{i}(t), \quad i = 1, \dots, N, \end{cases}$$
(6)

where $\hat{x}_i(t) \in \mathbb{R}^{n_i}$ is the observed state of $x_i(t)$ and $L_i \in \mathbb{R}^{n_i \times p_i}$ is the subsystem observation gain matrix.

The state observer of the global system (4), composed of N local observers, is given by

$$\begin{cases} \dot{x}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)], \\ \dot{y}(t) = C\hat{x}(t), \end{cases}$$
(7)

where $\hat{x}^{T}(t) = [\hat{x}_{1}^{T}(t) \ \hat{x}_{2}^{T}(t) \ \dots \ \hat{x}_{N}^{T}(t)]$ and $L = \text{diag}\{L_{i}\}$ represents the block diagonal of the observation gain.

The model reference of the *i*th subsystem can be designed by

$$\dot{x}_{ri}(t) = A_{ri}x_{ri}(t) + r_i(t) \tag{8}$$

where $x_{ri}(t) \in \mathbb{R}^{n_i}$ is the reference state vector of the *i*th subsystem, $A_{ri} \in \mathbb{R}^{n_i \times n_i}$ is the reference state matrix assumed to be asymptotically stable, and $r_i(t) \in \mathbb{R}^{n_i}$ constitutes a bounded reference input.

The model reference of the global system (4) is then given by

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{9}$$

where $x_r^T(t) = [x_{r1}^T \ x_{r2}^T \ \dots \ x_{rN}^T]$ is the reference state vector, $A_r = \text{diag}\{A_{ri}\}$ is the reference state matrix and

 $r^{T}(t) = [r_{1}^{T} \ r_{2}^{T} \ \dots \ r_{N}^{T}]$ is the bounded reference input of the global system.

On the other hand, the *i*th subsystem control law is expressed by

$$u_i(t) = K_i[x_{ri}(t) - \hat{x}_i(t)]$$
(10)

where $K_i \in R^{m_i \times n_i}$ is the local control gain matrix.

The model reference tracking control law for the global system (4) is as follows:

$$u(t) = K[x_r(t) - \hat{x}(t)]$$
(11)

with $K = \text{diag}\{K_i\}$ the decentralized control gain matrix.

Additionally, the local tracking error of each subsystem is given in the following form:

$$e_{ci}(t) = x_{ri}(t) - x_i(t).$$
 (12)

Moreover, the observation error related to the real and observed states is defined by

$$e_i(t) = x_i(t) - \hat{x}_i(t).$$
 (13)

The time derivative of (12) leads to

$$\dot{e}_{ci}(t) = [A_i - B_i K_i + \Delta A_i - \Delta B_i K_i] e_{ci}(t) - [B_i + \Delta B_i] K e_i(t) + [A_{ri} - A_i - \Delta A_i] x_{ri}(t) - \xi_i(t, x(t)) - \theta_i(t) + r_i(t).$$
(14)

The tracking error of the global system is described by

$$\dot{e}_c(t) = [A - BK + \Delta A - \Delta BK] e_c(t)$$

- [B + \Delta B]Ke(t) + [A_r - A - \Delta A]x_r(t)
- \xi(t, x(t)) - \theta(t) + r(t). (15)

The dynamics of the observation error is expressed by differentiating (13) as follows:

$$\dot{e}_{i}(t) = [A_{i} - L_{i}C_{i} + \Delta B_{i}K_{i}]e_{i}(t)$$

$$- [\Delta A_{i} - \Delta B_{i}K_{i}]e_{ci}(t)$$

$$+ \Delta Ax_{r}(t) + \xi_{i}(t,x(t)) + \theta_{i}(t).$$
(16)

The observation error of the global system can be expressed by

$$\dot{e}(t) = [A - LC + \Delta BK]e(t) - [\Delta A - \Delta BK]e_c(t) + \Delta Ax_r(t) + \xi(t, x(t)) + \theta(t).$$
(17)

Thereafter, the augmented system comprising the observation error (17), the tracking error (15) and the model reference (9), is given by

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{\theta}(t) + \tilde{\xi}(t, x(t)),$$
(18)

where



$$\begin{split} \tilde{A} &= \left[\begin{array}{cc} A - LC + \Delta BK & -\Delta A + \Delta BK \\ -BK - \Delta BK & A - BK + \Delta A - \Delta BK \\ 0 & 0 \\ \end{array} \right], \\ \tilde{B} &= \left[\begin{array}{c} AA \\ A_r - A + \Delta A \\ A_r \end{array} \right], \\ \tilde{B} &= \left[\begin{array}{c} I & 0 \\ -I & I \\ 0 & I \end{array} \right], \tilde{\xi}(t, x(t)) = \left[\begin{array}{c} \xi(t, x(t)) \\ -\xi(t, x(t)) \\ 0 \end{array} \right], \\ \text{and} \quad \tilde{\theta}(t) &= \left[\begin{array}{c} \theta(t) \\ r(t) \end{array} \right]. \end{split}$$

It is noted that the effect of the disturbance $\tilde{\theta}(t)$ can deteriorate the performances of the controlled system. To mitigate this effect, the following H_{∞} performance criterion can be used [8]:

$$\int_{0}^{t_{f}} e_{c}^{T}(t) Q_{c} e_{c}(t) dt + \int_{0}^{t_{f}} e^{T}(t) Q_{e} e(t) dt$$
$$= \int_{0}^{t_{f}} \tilde{x}^{T}(t) \tilde{Q} \tilde{x}(t) dt$$
$$\leq \eta^{2} \int_{0}^{t_{f}} \tilde{\theta}^{T}(t) \tilde{\theta}(t) dt, \qquad (19)$$

where η is a prescribed attenuation level, t_f reflects the final time of control, Q_c and Q_e are symmetric positive definite weighting matrices of control and observation for the global system. In addition, the matrix \tilde{Q} related to the overall closed-loop system (18) is defined by

$$\tilde{Q} = \begin{bmatrix} Q_e & 0 & 0\\ 0 & Q_c & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
 (20)

To propose new LMI conditions for the H_{∞} dynamic output-feedback decentralized control design method for nonlinear disturbed interconnected systems, the following Lemmas are required:

Lemma 1 [11]: For any matrices Γ , Δ and $\Lambda = \Lambda^T > 0$ of suitable dimensions, we have

$$\Gamma^{T}\Delta + \Delta^{T}\Gamma \leq \Gamma^{T}\Lambda\Gamma + \Delta^{T}\Lambda^{-1}\Delta.$$
(21)

Lemma 2 [12]: For real matrices M, N, Π, Λ, Ξ and a regular matrix *V* with suitable dimensions. It follows that

$$\begin{bmatrix} \Pi + N^{T} V^{-1} N & \Xi^{T} \\ \Xi & \Lambda + M V M^{T} \end{bmatrix} < 0$$

$$\Rightarrow \begin{bmatrix} \Pi & \Xi^{T} + N^{T} M^{T} \\ \Xi + M N & \Lambda \end{bmatrix} < 0.$$
(22)

Lemma 3 [13]: Consider a negative definite matrix Φ and a matrix Λ with a suitable size such as $\Lambda^T \Phi \Lambda \leq 0$. Then, $\exists \gamma \in \mathbb{R}$ so that the following inequality holds:

$$\Lambda^T \Phi \Lambda \le -\gamma (\Lambda^T + \Lambda) - \gamma^2 \Phi^{-1}.$$
(23)

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3. LMI CONDITIONS FOR ENSURING THE STABILITY AND H_{∞} PERFORMANCES OF THE AUGMENTED SYSTEM

In this section, it is investigated the problem of stability analysis and H_{∞} performance improvement of the decentralized feedback tracking controller design for (18). Hence, a positive-definite Lyapunov function is given by

$$V(\tilde{x}) = \tilde{x}^T P \tilde{x} , P = \begin{bmatrix} P_o & 0 & 0 \\ 0 & P_c & 0 \\ 0 & 0 & P_r \end{bmatrix}$$
(24)

where $P_o = \text{diag}\{P_{oi}\}, P_c = \text{diag}\{P_{ci}\}, P_r = \text{diag}\{P_{ri}\}$ are symmetric positive-definite Lyapunov matrices.

On the other hand, the augmented system (18) is stable in the sense of Lyapunov and the H_{∞} performance criterion is guaranteed for the attenuation level η if the following inequality is verified [14]:

$$\dot{V}(\tilde{x}) + \tilde{x}^T \tilde{Q} \tilde{x} - \eta^2 \tilde{\theta}^T \tilde{\theta} \le 0.$$
(25)

According to (18) and (24), the development of (25) leads to

$$\tilde{x}^{T} \left[\tilde{A}^{T} P + P \tilde{A} + \tilde{Q} \right] \tilde{x} + \tilde{\xi}^{T} P \tilde{x} + \tilde{x}^{T} P \tilde{\xi} + \tilde{\theta}^{T} \tilde{B}^{T} P \tilde{x} + \tilde{x}^{T} P \tilde{B} \tilde{\theta} - \eta^{2} \tilde{\theta}^{T} \tilde{\theta} \leq 0.$$
(26)

In order to rearrange the inequality (26) within a quadratic form, we use Lemma 1. Relying on (21), it follows that

$$\tilde{\xi}^T P \tilde{x} + \tilde{x}^T P \tilde{\xi} \le \tilde{\xi}^T \tilde{\xi} + \tilde{x}^T P P \tilde{x} \le \tilde{x}^T \left[M + PP \right] \tilde{x} \quad (27)$$

with

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\sum_{i=1}^{N} \alpha_i^2 I & -2\sum_{i=1}^{N} \alpha_i^2 I \\ 0 & -2\sum_{i=1}^{N} \alpha_i^2 I & 2\sum_{i=1}^{N} \alpha_i^2 I \end{bmatrix}.$$
 (28)

The inequality (26) can then be expressed by

$$\tilde{x}^{T} \left[\tilde{A}^{T} P + P \tilde{A} + P P + M + \tilde{Q} \right] \tilde{x} + \tilde{\theta}^{T} \tilde{B}^{T} P \tilde{x} + \tilde{x}^{T} P \tilde{B} \tilde{\theta} - \eta^{2} \tilde{\theta}^{T} \tilde{\theta} \leq 0$$
(29)

whih is equivalent to

$$\begin{bmatrix} \tilde{x}\\ \tilde{\theta} \end{bmatrix}^{T} \begin{bmatrix} \tilde{A}^{T}P + P\tilde{A} + PP + M + \tilde{Q} & P\tilde{B}\\ \tilde{B}^{T}P & -\eta^{2}I \end{bmatrix} \begin{bmatrix} \tilde{x}\\ \tilde{\theta} \end{bmatrix} \leq 0.$$
(30)

For the augmented system (18), if $P = P^T > 0$ is the unique solution of the inequality

$$\begin{bmatrix} \tilde{A}^{T}P + P\tilde{A} + PP + M + \tilde{Q} & P\tilde{B} \\ \tilde{B}^{T}P & -\eta^{2}I \end{bmatrix} < 0, \quad (31)$$

then it is stable within the Lyapunov framework and the H_{∞} performance is ensured for a level of attenuation η .

It is clear that (31) is not a tractable LMI, it can be developed according to (18), (20), (24) and (28) as follows:

$$\begin{bmatrix} X_{11} & X_{12} & P_o \Delta A & P_o & 0 \\ X_{12}^T & X_{22} & X_{23} & -P_c & P_c \\ \Delta A^T P_o & X_{23}^T & X_{33} & 0 & P_r \\ P_o & -P_c & 0 & -\eta^2 I & 0 \\ 0 & P_c & P_r & 0 & -\eta^2 I \end{bmatrix} < 0 \quad (32)$$

with

- $X_{11} = (A LC)^T P_o + P_o(A LC) + P_o P_o + Q_e + K^T \Delta B^T P_o + P_o \Delta B K;$
- $X_{12} = -K^T B^T P_c K^T \Delta B^T P_c P_o \Delta A + P_o \Delta B K;$

•
$$X_{22} = (A - BK)^r P_c + P_c (A - BK) + P_c P_c + Q_c + (\Delta A - \Delta BK)^T P_c + P_c (\Delta A - \Delta BK) + 2 \sum_{i=1}^N \alpha_i^2 I;$$

• $X_{23} = P_c (A_r - A) - P_c \Delta A - 2 \sum_{i=1}^N \alpha_i^2 I;$
• $X_{33} = A_r^T P_r + P_r A_r + P_r P_r + 2 \sum_{i=1}^N \alpha_i^2 I.$

It is worth pointing out that (32) is a BMI since it involves several cross coupling terms of P_o, P_c, P_r, L, K and η . To change this BMI into a standard LMI, let us part the terms containing uncertainties in (32). It yields

$$\Omega + \Delta \Omega < 0, \tag{33}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & -K^T B^T P_c & 0 & P_o & 0 \\ -P_c B K & \Omega_{22} & \Omega_{23} & -P_c & P_c \\ 0 & \Omega_{23}^T & \Omega_{33} & 0 & 0 \\ P_o & -P_c & 0 & -\eta^2 I & 0 \\ 0 & P_c & P_r & 0 & -\eta^2 I \end{bmatrix}$$

with

• $\Omega_{11} = A^T P_o + P_o A - C^T L^T P_o - P_o L C + P_o P_o + Q_e;$ • $\Omega_{22} = A^T P_c + P_c A - K^T B^T P_c - P_c B K + P_c P_c + Q_c + 2\sum_{i=1}^{N} \alpha_i^2 I;$

•
$$\Omega_{23} = P_c(A_r - A) - 2\sum_{i=1}^N \alpha_i^2 I;$$

• $\Omega_{33} = P_r A_r + A_r^T P_r + P_r P_r + 2\sum_{i=1}^N \alpha_i^2 I.$

and

with

•
$$\Delta \Omega_{11} = K^T \Delta B^T P_o + P_o \Delta B K;$$

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•
$$\Delta\Omega_{12} = -P_o\Delta A + P_o\Delta BK - K^T\Delta B^T P_c;$$

• $\Delta\Omega_{22} = \Delta A^T P_c + P_c\Delta A - K^T\Delta B^T P_c - P_c\Delta BK$

By using the norm-bounded uncertainty structure defined in (5) and Lemma 1 applied to the block diagonal matrices $\Delta\Omega_{11}$ and $\Delta\Omega_{22}$, $\Delta\Omega$ can be bounded as follows:

with

•
$$\Psi_{11} = \mu_1 K^T E_2^T E_2 K + \mu_1^{-1} P_o D D^T P_o;$$

• $\Psi_{22} = \mu_2 E_1^T E_1 + \mu_2^{-1} P_c D D^T P_c + \mu_3 K^T E_2^T E_2 K + \mu_3^{-1} P_c D D^T P_c.$

On the other hand, to transform the anti-diagonal uncertainty terms $\Delta\Omega_{12}$, $\Delta\Omega_{12}^T$, $P_o\Delta A$, $\Delta A^T P_o$, $P_c\Delta A$ and $\Delta A^T P_c$ into block diagonal matrices, we consider Lemma 2.

Hence, based on the use of the norm-bounded uncertainty structure defined in (5) and Lemma 2, the matrix $\Delta\Omega$ is then bounded by

with

- $\Phi_{11} = (\mu_1 + \mu_6)K^T E_2^T E_2 K + (\mu_1^{-1} + \mu_4^{-1} + \mu_5^{-1} + \mu_7^{-1})P_o DD^T P_o;$
- $\Phi_{22} = (\mu_2 + \mu_4)E_1^T E_1 + (\mu_2^{-1} + \mu_3^{-1} + \mu_6^{-1} + \mu_8^{-1})P_c DD^T P_c + (\mu_3 + \mu_5)K^T E_2^T E_2 K;$
- $\Phi_{33} = \mu_7 E_2^T E_2 + \mu_8 E_1^T E_1.$

Therefore, the inequality (33) becomes

$$\begin{bmatrix} \Omega_{11} + \Phi_{11} & -K^T B^T P_c & 0 & P_o & 0\\ -P_c BK & \Omega_{22} + \Phi_{22} & \Omega_{23} & -P_c & P_c\\ 0 & \Omega_{23}^T & \Omega_{33} + \Phi_{33} & 0 & P_r\\ P_o & -P_c & 0 & -\eta^2 I & 0\\ 0 & P_c & P_r & 0 & -\eta^2 I \end{bmatrix}$$

< 0. (35)

In order to rearrange the matrices involved in (35), let us first multiply respectively the left-hand side and the righthand side of this inequality by the full-rank matrices



$$\begin{bmatrix} \Omega_{11} + \Phi_{11} & P_o & | & -K^T B^T P_c & 0 & 0 \\ \hline P_o & -\eta^2 I & -P_c & 0 & 0 \\ \hline -P_c BK & -P_c & \Omega_{22} + \Phi_{22} & \Omega_{23} & P_c \\ 0 & 0 & \Omega_{23}^T & \Omega_{33} + \Phi_{33} & P_r \\ 0 & 0 & P_c & P_r & -\eta^2 I \end{bmatrix}$$

< 0. (36)

Additionally, the number of nonlinear terms can be decreased by multiplying the left-hand side and the right-hand side of (36) by diag{W, W, W, I, I} with $W = P_c^{-1}$, and by using the new parameters Y = KW and $Z = P_oL$. It yields

$$\begin{bmatrix} \underline{\Xi}_{11} & \underline{\Xi}_{12} \\ \underline{\Xi}_{12}^T & \underline{\Xi}_{22} \end{bmatrix} < 0, \tag{37}$$

where

$$\Xi_{11} = \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} T_1 & P_o \\ P_o & -\eta^2 I \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix} + \begin{bmatrix} (\mu_1 + \mu_6) Y^T E_2^T E_2 Y & 0 \\ 0 & 0 \end{bmatrix}$$
(38)

with $T_1 = A^T P_o + P_o A - C^T Z^T - ZC + P_o P_o + Q_e + (\mu_1^{-1} + \mu_4^{-1} + \mu_5^{-1} + \mu_7^{-1})P_o DD^T P_o.$

To bypass the problem of nonlinear terms in (38), one can use Lemma 3. Thus, (38) becomes

$$\Xi_{11} \leq -2\gamma \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix} - \gamma^2 \begin{bmatrix} T_1 & P_o \\ P_o & -\eta^2 I \end{bmatrix}^{-1} + \begin{bmatrix} (\mu_1 + \mu_6)Y^T E_2^T E_2 Y & 0 \\ 0 & 0 \end{bmatrix}$$
(39)

which is equivalent to

$$\Xi_{11} \leq \begin{bmatrix} -2\gamma W + (\mu_1 + \mu_6)Y^T E_2^T E_2 Y & 0\\ 0 & -2\gamma W \end{bmatrix} - \gamma^2 \begin{bmatrix} T_1 & P_o\\ P_o & -\eta^2 I \end{bmatrix}^{-1}.$$
 (40)

If the Schur complement formula is applied to the inequality (40), then it yields the inequality (41) at the top of the next page.

It should be noted that (41) is not a standard LMI. To overcome this problem, one can use the Schur complement formula a second time to transform (41) into LMI (42) with $\tilde{T}_1 = A^T P_o + P_o A - C^T Z^T - ZC + Q_e$ and $\Upsilon = -(\mu_1^{-1} + \mu_4^{-1} + \mu_5^{-1} + \mu_7^{-1})^{-1}I$.

On the other hand, based on the Schur complement formula, the matrix Ξ_{22} , defined in (37), is changed into the LMI (43) with $\tilde{T}_2 = WA^T + AW - Y^TB^T - BY + I +$ $(\mu_2^{-1} + \mu_3^{-1} + \mu_6^{-1} + \mu_8^{-1})DD^T$, $\tilde{T}_3 = A_r - A - 2\sum_{i=1}^N \alpha_i^2 W$ and $\tilde{T}_4 = A_r^T P_r + P_r A_r + \mu_7 E_2^T E_2 + \mu_8 E_1^T E_1 + 2\sum_{i=1}^N \alpha_i^2 I$.

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Besides, Ξ_{12} , involved in (37), can be written as

Finally, the inequality (37) is transformed into the following LMI:

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} < 0 \tag{45}$$

where Σ_{11}, Σ_{22} and Σ_{12} are defined respectively by (42), (43) and (44). The resolution of the LMI (45) leads to the calculation of the decentralized control gain $K = YW^{-1}$ as well as the decentralized observation gain $L = P_a^{-1}Z$.

To obtain an efficient tracking control with more excellently performances, the mitigation level η should be reduced as much as possible. Thereafter, the control design method is stated in the following minimization problem:

$$\begin{cases} \text{minimize } \eta^2 \\ \text{subject to } P_o = P_o^T > 0, W = W^T > 0, P_r = P_r^T > 0 \\ \text{and } (45). \end{cases}$$

$$(46)$$

The developed optimization problem (46) can be computed in a one-step procedure to concurrently extract the decentralized gains of the controller and the state observer.

4. APPLICATION OF THE PROPOSED APPROACH TO A POWER SYSTEM

This section aims to apply the proposed H_{∞} outputfeedback decentralized model reference tracking control design to a three interconncted machine power system. The studied process is presented in [15].

4.1. Power system model

It is noted that the power system involves three interconnected subsystems, where each subsystem is described by a fourth order nonlinear model including the governor/turbine dynamics. The model of each subsystem is expressed by the following state equation:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \xi_i(t, x(t)) + \theta_i(t) \\ y_i(t) = C_i x_i(t), \quad i = 1, 2, 3, \end{cases}$$
(47)

where $\xi_i(t,x(t)) = \sum_{j=1, j\neq i}^3 H_{ij}h_{ij}(x_i(t),x_j(t))$ reflects a vector of nonlinear functions realizing the interconnection among the *i*th and the *j*th subsystems, $\theta_i(t)$ is a vector of external disturbances, $u_i(t) = \Delta X_{ei}(t)$ is the input vector, $y_i(t) = \Delta \delta_i(t)$ is the observation vector and $x_i(t)$ is the state vector of the *i*th subsystem given by

$$x_i(t)^T = \begin{bmatrix} \Delta \delta_i(t) & \omega_i(t) & \Delta P_{m_i}(t) & \Delta X_{e_i}(t) \end{bmatrix}$$

with $\Delta \delta_i(t) = \delta_i(t) - \delta_{i0}$, $\Delta P_{m_i}(t) = P_{m_i}(t) - P_{m_i0}$ and $\Delta X_{e_i}(t) = X_{e_i}(t) - X_{e_i0}$.

The matrices of the *i*th subsystem are as follows:

$$A_{i} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{D_{ci}}{2H_{i}} & -\frac{\omega_{0}}{2H_{i}} & 0 \\ 0 & 0 & -\frac{1}{T_{m_{i}}} & \frac{K_{m_{i}}}{T_{m_{i}}} \\ 0 & -\frac{K_{e_{i}}}{T_{e_{i}}R_{i}\omega_{0}} & 0 & -\frac{1}{T_{e_{i}}} \end{bmatrix}, B_{i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{e_{i}}} \end{bmatrix}$$
$$C_{i} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

The interconnection parameters between two subsystems are given by $H_{ij} = \begin{bmatrix} 0 & -\alpha_{ij} & 0 & 0 \end{bmatrix}^T$, $\alpha_{ij} = \frac{w_0 E'_{ij} E'_{ij} B_{ij}}{2H_i}$ and $h_{ij}(x_i(t), x_j(t)) = \sin(\delta_i(t) - \delta_j(t)) - \sin(\delta_{i0} - \delta_{j0})$.

The generators and the transmission lines parameters of the studied power system are listed below [15].

- $\delta_i(t)$ Rotor angle for the *i*th machine, in rad;
- $\omega_i(t)$ Relative speed for the *i*th machine, in rad/s;
- $P_{mi}(t)$ Mechanical power for the *i*th machine, in pu;

 $X_{e_i}(t)$ Steam valve opening for the *i*th machine, in pu;

- H_i Inertia constant for the *i*th machine, in s;
- D_{ci} Damping coefficient for the *i*th machine, in pu;
- T_{mi} Time constant for the *i*th machine's turbine, in s;
- K_{m_i} Gain of the *i*th machine's turbine;
- T_{e_i} Time constant of the *i*th machine's speed governor, in s;
- K_{e_i} Gain of the *i*th machine's speed governor;
- R_i Regulation constant of the *i*th machine, in pu;
- B_{ii} Nodal susceptance between two machines, in pu;
- ω_0 Synchronous machine speed, in rad/s;
- E'_{qi} Internal transient voltage for the *i*th machine, in pu;
- δ_{i0} Nominal value of $\delta_i(t)$;
- $P_{m_{i0}}$ Nominal value of $P_{m_i}(t)$;
- $X_{e_{i0}}$ Nominal value of $X_{e_i}(t)$.

In the case of parameter variations, the nonlinear model (47) can be generalized by

$$\begin{cases} \dot{x}_{1}(t) = (A_{1} + \Delta A_{1})x_{1}(t) + (B_{1} + \Delta B_{1})u_{1}(t) \\ + H_{12}h_{12}(x_{1}(t), x_{2}(t)) + H_{13}h_{13}(x_{1}(t), x_{3}(t)) \\ + \theta_{1}(t) \\ \dot{x}_{2}(t) = (A_{2} + \Delta A_{2})x_{2}(t) + B_{2} + \Delta B_{2})u_{2}(t) \\ + H_{21}h_{21}(x_{2}(t), x_{1}(t)) + H_{23}h_{23}(x_{2}(t), x_{3}(t)) \\ + \theta_{2}(t) \\ \dot{x}_{3}(t) = (A_{3} + \Delta A_{3})x_{3}(t) + (B_{3} + \Delta B_{3})u_{3}(t) \\ + H_{31}h_{31}(x_{3}(t), x_{1}(t)) + H_{32}h_{32}(x_{3}(t), x_{2}(t)) \\ + \theta_{3}(t), \end{cases}$$
(48)

where ΔA_i and ΔB_i (i = 1, 2, 3) are assumed to be normbounded time-varying matrices having the same structure as (2) and $\theta_i(t), i = 1, 2, 3$, are external disturbances.

4.2. Simulation study

The 3-machine power system parameters are summarized in Table 1 [15].

Owing to [15], the parameters involved in H_{ij} are given by $\alpha_{12} = \alpha_{13} = -27.49, \alpha_{21} = \alpha_{23} = \alpha_{31} = \alpha_{32} = -23.10.$

On the other hand, the nonlinear interconnection functions of the studied power systems are detailed as follows:

$$\xi_{1}(t,x) = \begin{bmatrix} 0 \\ g_{12} \\ 0 \\ 0 \end{bmatrix}, \quad \xi_{2}(t,x) = \begin{bmatrix} 0 \\ g_{22} \\ 0 \\ 0 \end{bmatrix}$$

and $\xi_3(t,x) = \xi_2(t,x)$

with $g_{12} = H_{12}h_{12}(x_1(t), x_2(t)) + H_{13}h_{13}(x_1(t), x_3(t))$ and $g_{22} = H_{21}h_{21}(x_2(t), x_1(t)) + H_{23}h_{23}(x_2(t), x_3(t)).$

To test the robustness of the decentralized observerbased control approach against uncertainties, the normbounded matrices shown in (2) are illustrated by

$$\Delta A_1 = D_1 F_1(t) E_{1_1}, \ \Delta A_2 = D_2 F_2(t) E_{1_2}, \ \Delta A_3 = \Delta A_2,$$

$$\Delta B_1 = D_1 F_1(t) E_{2_1}, \ \Delta B_2 = D_2 F_2(t) E_{2_2}, \ \Delta B_3 = \Delta B_2,$$

with

$$\begin{split} D_1 &= \begin{bmatrix} 0 & 0 & 0 & 0.9 |\delta_1(t)|_{max} \end{bmatrix}^T, \\ F_1(t) &= \begin{bmatrix} 0 & -\frac{0.6 |\delta_1(t)|}{|\delta_1(t)|_{max}} & 0 & -\frac{0.6 |\delta_1(t)|}{|\delta_1(t)|_{max}} \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 0 & 0 & 0 & 0.9 |\delta_2(t)|_{max} \end{bmatrix}^T, \\ F_2(t) &= \begin{bmatrix} 0 & -\frac{0.6 |\delta_2(t)|}{|\delta_2(t)|_{max}} & 0 & -\frac{0.6 |\delta_2(t)|}{|\delta_2(t)|_{max}} \end{bmatrix}, \\ D_3 &= \begin{bmatrix} 0 & 0 & 0 & 0.9 |\delta_3(t)|_{max} \end{bmatrix}^T, \\ F_3(t) &= \begin{bmatrix} 0 & -\frac{0.6 |\delta_2(t)|}{|\delta_3(t)|_{max}} & 0 & -\frac{0.6 |\delta_2(t)|}{|\delta_3(t)|_{max}} \end{bmatrix}, \\ E_{1_1} &= E_{1_2} = E_{1_3} = I_4 \end{split}$$

and $E_{2_1} = E_{2_2} = E_{2_3} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$.

It is worth noting that $\delta_i(t) = \frac{1}{T_{ei}} - \frac{1}{T_{ei} - \Delta T_{ei}}$ is the parametric variation characterizing the time constant of the *i*th machine's speed governor with $|\delta_i(t)| \leq 1$.

Table 1. Parameters of the three interconnected machines.

Parameters	Machine 1	Machine 2	Machine 3
$x_T(pu)$	0.129	0.11	0.11
$x_d(pu)$	1.863	2.36	2.36
$x'_d(pu)$	0.257	0.319	0.319
H(s)	4	5.1	5.1
$D_c(pu)$	5	3	3
$T_m(s)$	0.35	0.35	0.35
$T_e(s)$	0.2	0.2	0.2
R(pu)	0.05	0.05	0.05
K_m, K_e	1	1	1
$\omega_0(rad/s)$	314.159	314.159	314.159
$x_{ij}(pu)$	$x_{12} = 0.55$	$x_{23} = 0.6$	$x_{31} = 0.53$

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To demonstrate the performances of the proposed approach, the model reference parameters are given by

$$A_{ri} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -100 & -101 & -102 & -103 \\ 0 & 0 & -10 & -1 \\ 0 & 0 & -1 & -10 \end{bmatrix},$$
$$r_i = \begin{bmatrix} 0 \\ 260\cos(0.75t) \\ 0 \\ 0 \end{bmatrix}.$$

The outcomes of the optimization problem (46), applied to the power system model (48) with $Q_e = 10I_{12}$ and $Q_c = 10I_{12}$, are focused upon the decentralized control gain matrix, $K = \text{diag}\{K_i\}, i = 1, 2, 3$ such as

$K_1 = [$	6.5862	2.6599	14.1970	2.2455]	,
$K_2 = [$	7.1526	3.0956	13.2886	2.1526]	,
$K_3 =$	7.1526	3.0956	13.2886	2.1526	,

and the decentralized observation gain, $L = \text{diag}\{L_i\}, i = 1, 2, 3$ with

$$L_{1} = \begin{bmatrix} 8.5198\\ 22.5382\\ 1.3224\\ -3.9669 \end{bmatrix}, L_{2} = \begin{bmatrix} 7.8117\\ 16.5221\\ 1.1348\\ -5.5096 \end{bmatrix},$$
$$L_{3} = \begin{bmatrix} 7.8117\\ 16.5221\\ 1.1348\\ -5.5096 \end{bmatrix}.$$

The simulation results of the H_{∞} decentralized tracking control design are depicted in Fig. 1, in which the evolution of the rotor angle variation $\Delta \delta_i$, the relative speed w_i , the mechanical power variation ΔP_{mi} and the steam valve opening variation ΔX_{ei} and their corresponding reference models are simulated despite the nonlinear interconnection terms, the uncertain parameter variation and the external disturbances $\theta_i(t) = \begin{bmatrix} 0 & 0.7 \sin(2t) & 0 & 0 \end{bmatrix}^T$ applied to w_i , i = 1, 2, 3, which constitutes an amplitude about 40% of the tracking trajectory. These graphics show the remarkable performances of the proposed control scheme.

Furthermore, to test the high performances of the designed control approach, strong external disturbances illustrated by $\theta_i(t) = \begin{bmatrix} 0 & 1.75 \sin(2t) & 0 & 0 \end{bmatrix}^T$ with an amplitude about 100% of the tracking trajectory, are applied to the studied power system. The simulation results obtained with the same initial conditions used in the foregoing case, are depicted in Fig. 2. It is plainly shown that the power system does not have an unstable behavior. In fact, all the state variables track effectively the reference





Fig. 1. Tracking control of the 3-machine power system towards a disturbance of amplitude 40% applied on w_i , i = 1, 2, 3.



Fig. 2. Tracking control of the 3-machine power system towards a disturbance of amplitude 100% applied on w_i , i = 1, 2, 3.

5. CONCLUSION

In this work, the design of robust decentralized tracking control has been addressed for a class of interconnected systems. The asymptotic stability and the H_{∞} control synthesis of the closed-loop system have been developed as an optimization problem in terms of LMI constraints. The proposed control scheme has been efficiently solved by a one-step procedure to evaluate concurrently the decentralized control and observation gains. The simulation results have demonstrated the outstanding performances of the developed approach applied to a 3-strongly interconnected machine power system in spite of parameter uncer-

tainties and exogenous disturbances. Thereby, the elaborated LMI optimization conditions can be contemplated as significant improvements of precedent works dealing with this subject. Moreover, the developed approach can be extended to nonlinear interconnected systems with timevarying delays, which could be my future work.

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